

# The Joy of Quantum Physics: A User's Manual

Michael A. Morrison  
Department of Physics & Astronomy  
University of Oklahoma

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To Neal Lane,  
former research advisor and, ever since, a valued friend and mentor;  
to the many students, graduate and undergraduate,  
who have taught me so much about how to help others learn;  
and, most of all, to my wife Mary,  
without whose support this book would not exist.

Another damned thick book:  
always scribble, scribble, eh Gibbon.

—Comment to the historian Edward Gibbon  
from his patron the Duke of Gloucester  
on the publication of the second volume  
of Gibbon's *Decline and Fall of the Roman Empire*.

Everything should be made as simple as possible,  
but not simpler.

—Albert Einstein

In a contemplative fashion,  
And a tranquil frame of mind,  
Free from every kind of passion,  
Some solution let us find.  
Let us grasp the situation,  
Solve the complicated plot—  
Quiet, calm deliberation  
Disentangles every knot.

—*The Gondoliers*,  
by W. S. Gilbert and Sir Arthur Sullivan

It is characteristic of all deep human problems  
that they are not to be approached without  
some humor and some bewilderment—  
science is no exception.

—*Disturbing the Universe*  
by Freeman Dyson

Man, wow, there's so many things to do, so many thing to write  
How to even *begin* to get it all down and without modified restraints and all  
hung-up on like literary inhibition and grammatical fears. . .

‘Whooooo, here we go!’

—*On the Road* by Jack Kerouac

# Many thanks!

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If I had been so lucky as to have a steady brother  
Who could talk to me as we are talking now to one another—  
Who could give me good advice when he discovered I was erring. . .

—*Ruddigore*, by W. S. Gilbert and Arthur Sullivan

I'm particularly happy to acknowledge members of the Department of Physics and Astronomy at the University of Oklahoma who gave generously of their time and knowledge to the improvement of this book. Bruce Mason and Greg Parker taught (heroically) from early versions and let me extract reactions, suggestions, and gripes from their students. Both Bruce and Greg as well as Eric Abraham, Tom Miller, Jim Shaffer, and Kieran Mullen, critiqued drafts of material about which their expertise exceeds my own, keeping those parts accurate and suggesting improvements that make me look smarter than I am.

Three of my teachers at Rice University exerted so great an influence on me that I and this book owe them an uncountable debt: G. King Walters, Thomas L. Estle, and, especially, my doctoral advisor Neal Lane. By repeatedly challenging me to *think*, rather than merely derive, encode, and calculate, these teachers opened my eyes to the beauty and excitement of physics and propelled me on a life-long quest for insight whose current way station is this book. Later, other friends and colleagues—notably Keith MacAdam, Bill McCurdy, Rob Robson, Andy Feldt, Lee Collins, James Cohen, Robert Compton, Steve Buckman, David Norcross, and Barbara Whitten—continued to deepen my understanding of physics.

And thanks to Limpett, Hrothgar, Asphalt, and Muon. Cats are great company for a writer/teacher: they pay attention, don't argue, and never complain about their grade.

But my greatest debt is to the hundreds (thousands?) of students with whom I've worked in physics courses. Students at the University of Oklahoma, where I've spent almost my entire career, are wonderful, generous people. From them I learned much of what I know about how to help people learn. While this book has benefited from thousands of comments and suggestions made by students in quantum courses, the manuscript was particularly influenced by two graduate students, James Dizikes, and Stéphane Valladier, who worked it over just prior to my closing the book (so to speak) on it. While I didn't adopt all of the suggestions they and others offered, I considered every one and implemented many.

In a similar vein, I want to acknowledge a small handful of other books that influenced this one in an almost subliminal way. (The myriad books that influenced this one directly are cited in footnotes, Suggested Readings, and the Bibliography.) With one exception, these books did not provide direct grist for *The Joy of Quantum Physics*. Rather, because of their pedagogical clarity, care, and precision, they have served as perpetual fountains of insight and inspiration: *Thermodynamics and an Introduction to Thermostatistics* (Second edition) by Herbert B. Callen (New York: Wiley, 1985); *Quantum Mechanics* by Claude Cohen-Tannoudji and Bernard Diu and Franck Laloë (New

York: Wiley-Interscience, 1977); *Scattering Theory* by John R. Taylor (New York: Wiley, 1972); and *Visual Quantum Mechanics* and *Advanced Visual Quantum Mechanics* by Bernd Thaller (New York: Springer, 2000 and 2005, respectively).

This book would not exist in its current form were it not for the  $\text{\LaTeX}$  and *Mathematica*. I calculated all original numerical results using either (cited) on-line sites, applets, and programs or, more commonly, using *Mathematica*. And, except for a few recalcitrant expressions, whose complexity exceeded even *Mathematica*'s capabilities, I checked all but the simplest expressions and equations with *Mathematica*. To minimize typos, I directly exported results from *Mathematica* into  $\text{\LaTeX}$ . This doesn't mean that every number, expression, and equation in this book is correct; but it does mean that *more* are correct than had I done everything by hand or copied from other sources, many of which contain their own errors. It also means that you can replicate, play around with, and extend almost everything in this book if you know how to use *Mathematica* or an equivalent software package—an investment of time and energy I emphatically recommend. Among the many *Mathematica* gurus who came to my rescue from time to time, I particularly thank Mark Caprio, whose remarkable package `LevelScheme` enabled me to painlessly prepare figures I previously could not prepare at all.

Finally, special thanks to the many people—mostly students, though not all in colleges or universities—who have sent me letters or emails about my previous book *Understanding Quantum Physics*. Your communiques, all of which I kept, sustained me through dark days when I considered abandoning this project. Without your feedback the book you hold in your hands would not exist.

Ultimately, this book is for you, readers and students—not just during whatever course or period of time you intensively study it, but throughout your career. Whether you become (or are) a professional physicist, you who undertake to journey through a book like this one are blessed with a curiosity about nature that will last throughout your life. I hope *The Joy of Quantum Physics* serves as a trustworthy travel guide during your adventures in the microverse.

# Before we begin

Version 7.10: February 26, 2008

Let's talk physics. I invite you to sit down with me several times a week for the next several months to explore one of the most fascinating topics in science: quantum physics. Regular meetings in a coffee shop or brew pub being, alas, impractical, I offer instead this book.

## Content and connections

What's in this book? You can read the table of contents, so I'll not reiterate it here. Instead, here's an overview.

Building on a foundation of introductory quantum mechanics at roughly the level now (2008) taught in most sophomore/junior "modern physics" courses, *The Joy of Quantum Physics* develops the *nonrelativistic quantum physics of matter* in four stages: (1) a single spinless particle in two and three dimensions; (2) spin and its consequences; (3) systems with more than one identical particle; and (4) ways to solve problems approximately using analytic techniques. (To my regret, practical limitations forced me to jettison *computational* solution of quantum problems.)

As important as the topics in the table of contents are a handful of *themes* that knit these topics together. These themes appear in the first few chapters, then reappear and are elaborated throughout the book. In them resides the depth, majesty, and beauty that is the essence of quantum physics.

- (1) *The unity of quantum physics.* Most newcomers are prone, as I was, to see quantum mechanics as a collection of vaguely related topics that happen to appear between the covers of a book, like pieces of an incomplete jigsaw puzzle in a box. If allowed to fester, this viewpoint will make learning quantum mechanics unnecessarily arduous and frustrating.
- (2) *Symmetry and its consequences.* Apart from a few architects, most people find great beauty in symmetry. The study of nature by anyone is enriched by an awareness and appreciation of symmetry. The study of nature by a scientist is also *simplified* by symmetry. The symmetry of a system is manifest in fundamental *invariance principles*. These principles yield deep insights into a system and how it relates to other systems and will recur repeatedly as we work together.
- (3) *Models.* Nature is sufficiently complicated that understanding, whether qualitative or quantitative, requires the use of models. A model is a *story* scientists tell about how we *think* nature works. Unlike a novel or a play, a model is ultimately critiqued by nature: either the model's predictions conform to experiment or they don't. If they do, we attribute a provisional veracity to the story, always keeping in mind that subsequent experiments may require substantial revi-

sions. Models rely on physical insight into the *qualitative essence* of systems and their states, a type of insight this book strives to develop.

- (4) ***Generic properties of quantum states.*** Wave functions for many systems satisfy a few properties simply because they must solve the Schrödinger equation and certain boundary conditions. Because these properties do *not* depend on how we model the system—in this sense they’re “generic”—they serve as powerful guidelines for constructing models and for determining the validity of results of analysis or calculations. These properties are simple to remember, powerful tools for ***brainstorming*** on a quantum system, a skill we’ll work hard to master.
- (5) ***An approach to thinking about physics and working physics problems.*** At some level every text actualizes the author’s philosophy of physics—a personal way of conceptualizing physical ideas and of translating these ideas into mathematics. In most texts, this way of thinking is implicit, below the surface of words, equations, and so forth. I’ve tried to bring my way of thinking about physics onto the surface of the text—not because I think my way is necessarily the “best” way, but because I think seeing your author’s way of thinking will help you to develop *your own individual style as a physicist*, which should be one of your long-term goals.

### Who are you?

I designed this book for a variety of users—those trying to learn quantum physics on their own, those teaching or taking a course in quantum physics, and those who are just curious and adventurous. How to best approach this book depends on your goals, background, and the context in which you’re working with it.

### What do you know?

Since the publication in 1990 of my previous book *Understanding Quantum Physics*, much of “introductory quantum mechanics” has diffused downward through the physics curriculum. Topics that used to appear only in graduate courses now show up in junior and senior courses. Topics previously restricted to upper-level undergraduate courses now show up in sophomore courses on modern physics or even in second-semester freshman physics. This diffusion has exacerbated a problem all textbook authors face—the problem of background: Where to start? What to assume? What to leave out?

Rather than itemize here the quantum background you need for this book, I’ve prepared an appendix, “Quantum Physics’ Greatest Hits,” that reviews and summarizes this stuff. You need not be a master of *everything* in this appendix; but if *a lot* of it is unfamiliar, you should spend some quality time with a lower-level quantum book. (I can recommend one.) You also need to know the physics of waves, classical mechanics and electromagnetism, calculus, and how to solve differential equations. . . all at the freshman or sophomore level.

## A collaborative effort

In many respects, this book is the result of a (very) long collaboration between me and hundreds (thousands?) of students. Many of the Appendixes, optional sections, Asides, and footnotes—*about which please read the following “Note to the User”*—resulted from interactions with students. Here’s an example: students repeatedly complained that they had trouble relating what they were reading in the “official” text for the course to what they found in other recommended books, the study of which I passionately recommend. So this book contains many footnotes that address widespread conflicts in terminology (jargon) and notation in the pedagogical literature. Here’s another example: if I occasionally seem excessively emphatic or a bit repetitious, the reason is that year after year after year students have tripped over the same small but vital points, the ones I thus emphasize.

All these students also served as more-or-less willing guinea pigs for the exercises and problems I developed for this book. Studying students’ work and comments led me to completely revise or toss out lots of problems. Those problem that made the final cut, all of which have been class tested at least once, come with a “difficulty rating scale” (from one to four stars, one star being the least challenging) based not on my opinion but on student input.

## Extras

From the outset, I planned this book to be not only a textbook for immediate study but also a long-term resource, a book you’ll use for the rest of your career. These “extras” comprise a lot of stuff you’re *not* supposed to read the first time through (see the following “Note to the User”). For instance, each chapter comes with specific *citations* to other sources; some present a different point of view; others additional examples or applications to other fields of physics or technology. Still others extend a topic to a more advanced level. Moreover, many chapters contain *optional (starred) sections*, *end-of chapter complements*, and various of *Asides* and *footnotes*. The book itself comes with lots of *appendices*. (With a nod to the excellent graduate text by Cohen-Tannoudji et. al, I call end-of-chapter appendixes “complements.”) At the back of the book you’ll find, in addition to the usual tables of units, constants, conversion factors, and math stuff, appendixes that contain extended discussions of atomic units; summaries of properties of relevant special functions such as associated Legendre functions, associated Laguerre polynomials, and spherical harmonics; detailed solutions by calculus of some key equations that, in the text proper, are solved by recourse to special functions; introductions to relevant, interesting and (slightly) more advanced topics such as the Runge-Lenz vector; and, perhaps most useful, a summary of the seven problem-solving strategies I’m trying to help you master.

## An approach to physics pedagogy

Like its predecessor *Understanding Quantum Physics*, this book embodies my conviction that the best preparation for future study and use of a topic is a solid, in-depth understanding *of its most essential ideas in the simplest possible context*—the simplest possible models in the fewest possible dimensions. (That’s why, for example, we study orbital angular momentum first in two dimensions, then in three.)

Furthermore, I consider it important for students to at least glimpse the human side of physics. Quantum mechanics did not appear by magic or fiat. Physicists did not always greet its coming with open arms. Nor were its creators demigods. Physicists—even great physicists like the creators of quantum mechanics—are just people. Above all they are *professionals*—smart, very well-trained, serious about and committed to what they do. A very few were geniuses; most of us aren’t. Most of us accomplish whatever we accomplish by working hard, being systematic and (very) careful,



and nurturing the curiosity about nature that drove us into physics in the first place. To provide a modicum human context, I've included a few historical Asides.

Learning quantum mechanics requires hard work. But it need not induce anguish. I mean the title of this book literally: studying quantum physics should be joyful. Joy and the hard work of learning are not mutually exclusive... if you think they are, you might consider a career other than physics. For a physicist, as for any professional, study doesn't end with a bachelor's, master's, or PhD. A pleasure and privilege of being a scientist is that one remains a student of nature throughout one's entire career. (It still amazes me that I get paid to do this.) The joy of really *understanding* even a tiny part of how nature works justifies, for us, the hard, often tedious work required. I wrote this book to share with you this sensibility and, insofar as I can, to help you tap your innate ability to experience the joy of quantum physics.

# To the user

Version 7.10: February 26, 2008

## How to learn most effectively from this book

This book is like an archeological dig—only instead of pots and shards its treasures are information and insight. To learn effectively from this book, you must appreciate its “vertical structure.” Information comes in layers. In each chapter there is a *surface layer*—a story, if you will—that is presented in the primary text (regular typeface, no special indentation). At the start of each chapter you’ll find a summary of this story: a a brief overview and a detailed table of contents. Beneath this surface are layers that flesh out the story—that illustrate it, extend it in other directions, clarify plot developments, and so forth. This stuff is contained in Asides, Complements, Examples, in-text Questions, and footnotes. This structure explains why you should approach this book a bit differently from others you’ve studied. The single most important piece of advice I can offer is:

◆ **Rule: The first time or two you study a chapter, study only the surface layer:**

- ▶ Don’t read the Asides.
- ▶ Don’t read the optional (starred) sections.
- ▶ Don’t read the footnotes.
- ▶ Don’t read the Complements at the end of each chapter.
- ▶ Don’t read the in-text questions (but work them next time!).

**Your goal the first time through is to grasp the big picture, the arc of the story the chapter tells.**

Each Aside, Complement, optional section, in-text question, example, exercise, and problem comes with a label so you can tell at a glance what it’s about. Most footnotes are labeled. When first reading a chapter you should skip most footnotes—except, perhaps, those labeled *Notation*. These contain brief comments designed to clarify possible confusion concerning notation—which in physics is an *endless* source of possible confusion. Footnotes labeled *Jargon* seek to clarify confusion you may encounter when reading other sources—which you can find via footnotes labeled *Read On*. And so forth.

Each chapter contains other study aids—most notably a “User’s Guide” at the end. In addition to a summary of key equations and tables of useful results, each User’s Guide contains a **table of notation** introduced in that chapter. *Skim this table before you study the chapter and refer to it if you forget the meaning of a symbol.* Several sections end with lists of **key points**, summaries

of the most important items in the chapter. And, of course, each chapter concludes with exercises and problems. (I'll spare you the usual exhortation to work lots of problems. Surely you know by now that you'll learn about as much about physics by *reading* a physics text as you'd learn about swimming by reading a pamphlet about lifesaving.) **Exercises** come in three varieties: (1) review questions; (2) algebraic practice with key tools introduced in the chapter; and (3) modest numerical calculations designed to help you become familiar with the order of magnitude of key quantities introduced in the chapter. **Problems**... well, by now you know what a "problem" is. Most problems carry a rating of from one to four stars based on input from students (the more stars, the more challenging the problem). Since most problems "talk you through" the solution, you'll benefit from reading the problems you don't have time to work—though not as much as if you work them.

I use the following typeface conventions: *italic* for emphasis, **bold-faced italic** for the *first* introduction of a new term and for major information about the term, and pure **bold face** for emphasis of the use of a technical term. Every such term is defined explicitly near its first appearance; if you need to review the definition, head for the index.

### Attaining mastery

Here are a few other tips on how to most effectively approach each chapter:

- *Don't try to read an entire chapter at one sitting.* Give yourself a break. Divide the chapter into accessible chunks of material and study it over several sessions. This tactic gives your brain time between study sessions to assimilate what you've learned. Set aside time to re-read each chunk of material at least once; every re-reading will bring the material into sharper focus.
- On readings *after your first*, think through or work out the in-text questions labeled **Try This**. Merely reading this book may be interesting and even entertaining but it won't net you understanding, insight, or skills. To attain those you must *participate as you read*.
- In addition to figuring out the in-text questions, read actively in every way you can. Keep paper by your side and a pencil in your hand. Take notes. Work out details. Formulate and, if possible, answer your own questions about what you read. Most important, summarize and synthesize what you read *in your own words*.

### Doing physics

To learn how to think about physics you have to learn how to solve problems. Professional physicists approach problems differently than the way most students approach problems: they use a handful of *problem-solving strategies*—general ways to approach a problem that make its solution less arduous, more likely correct, and more generative of insight. By applying these strategies over and over, to a diverse spectrum of problems, professional physicists absorb these strategies, turning them into *habits for effective problem solving*.

You probably already know some problem-solving strategies. Good freshman physics books illustrate many. But problems you solve as a junior and senior—and will solve as a professional physicist—rarely resemble problems you solved as a freshman. Real physics problems often require some formulation; usually require mathematical analysis, approximations, and/or computations; and, alas, lack answers in the back of the book. A key part of my agenda is to help you develop problem-solving strategies that will serve you during and after your study of this book.

These strategies—which I've summarized in an appendix called (with apologies to Stephen R. Covey) "***The Seven Habits of Highly Effective Problem Solvers***"—are by no means the

*only* useful strategies for problem solving. But they work: they minimize error, minimize time and effort, minimize algebra and calculus, and enhance insight.

**On your mark, get set. . .**

I hope you enjoy working with this book. I hope you learn a lot from it. I hope it becomes a permanent and *useful* part of your working library. I hope you'll let me know what you think and how I could improve this book should I revise it. Above all, I hope you leave our time together eager to explore further quantum physics and its applications. The microverse is endlessly fascinating—so rich that anyone can find topics intriguing enough to last a lifetime.

—Michael A. Morrison  
Norman, Oklahoma  
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