Appendix A

The Seven Habits of Highly Effective Problem Solvers

> ... it's so simple,
> sublimely simple.
> If you learn not just to see but to observe.
> Put your brain to work, not just the optic nerve
> If you put your mind to use,
> You will find the most abstruse
> Becomes so simple ...
>
> —Sherlock Holmes, in
> *Baker Street: A Musical*

§ A.1 Introduction

A student who undertakes to study college-level physics brings with him or her a bunch of *problem-solving techniques* most people pick up in high school. These techniques may work for simple plug-and-grind problems but are useless for the more interesting problems one encounters in quantum mechanics. By now, many of these habits are almost second nature. I also believe that many of these habits are counterproductive, and that you'll have a lot more success and a lot more fun if you change them . This appendix describes one approach to more thoughtful, enjoyable problem solving. This is filled with examples of how to use these techniques, so you will find here just a summary.

Three premises are inherent in this approach:

1. You already know much more than you realize; your problem is accessing and using that knowledge effectively.
2. Knowledge is *hierarchical and cumulative*, and the most effective ways of accessing and using knowledge take these features into account.
3. Since problem solving is a lot of work. So should be as fruitful, painless and even fun as possible.

These principles constitute an approach that maximizes fun and insight and minimizes algebra, error, and tedium.

THE SEVEN PRINCIPLES

- ***The Principle of the Staged Assault***
- ***The Principle of Brainstorming***
- ***The Principle of Modeling***
- ***The Principle of Pattern Seeking***

- ***The Principle of Back to Basics***
- ***The Principle of Least Algebra***
- ***The Principle of Alert Awareness***

§ A.2 Implementation Strategies

A.2.1 The Principle of the Staged Assault

Approach problems systematically, strategically, and in a spirit of play. Approach problems in three stages:

1. **Feed your mind, then give it a break.**
2. **Brainstorm the problem.**
3. **Go for it.**

- When you decide to (or are assigned to) work a problem, *read it immediately.* Just read it: don't try to solve it; don't fret about it. But read it with your mind, not just your eyes.
- If time permits, put the problem aside for a day or so and go about your life while your mind digests and works on the problem. Then take out the problem and brainstorm (see below). Then *and only then* take out your calculator, tables, text, scratch paper, and start work.
- *Don't put yourself under unnecessary stress by letting the problem sit around too long.* (Procrastination isn't immoral, but neither is it cost effective.) Start work early. Give yourself a break: Work on problems during frequent brief sessions, rather than cramming everything into one marathon slugfest with the problem.
- Always be willing to put a problem aside for a while, to return to it later. You'll be surprised what your mind figured out while you were doing other things!

A.2.2 The Principle of Brainstorming

Plan. Think about what you're going to do. Then do it.

Brainstorming is easy. You're not trying to work the problem. You're not trying to get the right answer. You're not doing calculations. You're not rummaging through books looking for relevant equations. You're just playing around with the ideas raised by the problem. Once you're into this approach, you'll find that brainstorming won't take long—but I guarantee that brainstorming will save you huge amounts of time and energy when you get into actually working the problem!

A few guidelines are essential to successful brainstorming.

- Jot down as many ways to approach the problem (or parts of it) as you can think of.
- Don't worry at all about practical issues (can I do this? how hard would it be?)
- Brainstorming should involve few or no equations and few or no calculations; rather, you're trying to wrap your mind around the problem. Once you've done that, the equations and calculations will follow—and be lots less trouble than if you'd jumped right to them!

Key questions for a brainstorming session might include

1. What is the problem really asking for? That is, what are the key questions? Do I understand the problem? To check, try to restate it in your own words *without referring to the original statement.*

2. What are the physical principles involved in the problem situation? (This does not mean "what equations do I use?")
3. Can I simplify the problem with approximations or by modeling the system (see below)? Can I think of more than one such approximation or model?
4. How can I express my understanding of the problem in sketches? Draw lots of *simple, rough, qualitative sketches*—as many as you can think of—to illustrate or clarify the situation posed in the problem.

♦ **Tip:** *Try to keep your brainstorming notes informal but neat enough that you can read them. Prepare a clear* written *list—of about half a page—of the key elements of the problem (quantities, physical principles, data supplied, etc.). When you start work on the problem, this list is your guide, your map out of any swamps of deadly algebra into which you might sink.*

♦ **Tip:** *Once you've come up with one way to approach the problem, stop. Reconsider. Ask yourself, "What is another way to solve this problem? Can I think of several ways?" If so, jot them down. Then ask, "which way seems easiest and clearest and most direct?"*

A.2.3 The Principle of Modeling

Depending on the nature of the problem you're interested in, a vital part of your preliminary brainstorming may be designing a **model** of the physical system involved. The model follows naturally from your ideas about what physical principles and quantities are most important for the problem–ideas you developed in your brainstorming session. Key questions for model building include

1. *Precisely* what quantities do I need to evaluate?
2. How accurately do I need to know these quantities?
3. What physical features of the system will affect these quantities?[1]
4. Which of these features will *least* affect the quantities I want?
5. Based on this analysis, which features of the physics *must I retain* to get the accuracy I desire in the quantities I'm interested?

In your first stab at a model, jettison every feature of the physics that didn't meet the criteria you've established. That is, once you've answered these questions, *throw out as many of the complicating physical aspects of the problem as possible.* What's left is your model.

♦ **Tip:** *Don't haul out the equations too soon! Only when you can write down a* verbal *description of your model (perhaps with rough sketches) and identify the approximations inherent in the model (with justification)... only then are you ready to translate your ideas into mathematics (equations and data).*

[1]For example, in many such problems you can get insight into this matter by thinking about the physical meaning of the terms in the potential energy.

A.2.4 The Principle of Pattern Seeking

> **You know a lot of stuff. Use what you know. Familiarity Breeds success. Before you start work, try to relate what you know to what you want to know. Seek patterns over and over as you work.**

At several intervals during your work on the problem, as yourself questions like the following.

- What is *the main thing* this problem is asking? What do I already know about this thing?
- What *patterns and relationships* do I see in this problem?
- *How does this problem resemble problems I've already solved?* That is, seek analogies between problems you've solved and the one you want to solve. *Then list the similarities and differences between the two problems!* Often this step alone will trigger your mind to find the road to an efficient, effective approach.
- How can I adapt known results from other physical systems to the context of this problem?

♦ **Tip:** *If you're having trouble formulating the problem you've been asked to solve, make up and solve another problem! That is, devise a simpler problem that involves the same physical principles and quantities. The simpler the better! Then seek patterns between the simpler problem(s) and the one you want to solve.*

Now that you've completed the three preliminary stages, we're ready to start work. The following strategies "interact." That is, as you work the problem you'll move back and forth among them. As you work, repeatedly refer to and update your brainstorming notes, model design, and related problems and patterns! They're your guide to efficient, effective problem solving!

A.2.5 The Principle of Back-to-Basics

> **Whenever you feel yourself getting lost or swamped, stop, look at your list of key elements. Try to navigate back to the basic physical concepts, quantities, and questions of the problem. This may mean temporarily abandoning whatever tributary you've gone down, but that's okay; you can always return later.**

As you dig into even a moderately complicated problem, you'll probably find yourself moving ever deeper into a morass of symbols, explicit equations, numbers, units, and algebra. What do you do if you get really lost? We've all been there—and you can pull yourself out. Usually people get lost in solving problems for one of three reasons:

1. They don't know where they want to go and haven't a clue how to get there. (This won't happen to you, because you've brainstormed the problem.)
2. They're using equations that are more complicated than they need to be. (This won't happen to you if you keep your attention focused on the physical principles involved, rather than just trying to find any old equation that relates the physical quantities given in the problem.)
3. They're trying to do too many things at once (juggle equations, solve equations, plug numbers into equations, etc.)

You can save yourself lots of work and even more grief, if you'll follow these guidelines. When you feel yourself loosing your way, the first thing to do is

$$\boxed{\textbf{STOP!}}$$

Take a deep breath. Go get a cup of coffee (or whatever). Now ask yourself

1. Am I following one of the approaches I laid out in my brainstorming, or have I gone off on a detour? If the latter, is this a productive detour, or am I just goofing around?
2. Do I *really* have to use these complicated equations? Are there more variables in these equations than appear in my key list? Can I find a simpler equation that mathematically articulates the physical principles I've identified?

3. Am I trying to do several things at once?

♦ **Tip:** *The last point is worth emphasizing. Particularly when you're facing a new type of problem or one that is otherwise unfamiliar,* approach the problem in stages *(see above). Select the relevant equations. Relate these equations. Try to simplify them if necessary. Plug explicit expressions into them (see below). Plug numbers in, if necessary. Interpret the equations physically and/or graphically.* But never never do all this at once.

♦ **Tip:** *Use your brainstorming notes as you work. Update and annotate new quantities you introduce, equations involving these quantities, and intermediate or auxiliary results. Keep it neat. You need this map! It may save your life (metaphorically speaking, of course).*

A.2.6 The Principle of Least Algebra

We now come to the most important principle of effective problem solving. The basic idea is simple: There is no problem which, however difficult, cannot be made more difficult if you approach it right.

> **Clean up your act! Before you start manipulating equations, do everything you can to make them look as simple as possible. As you work, repeatedly try to simplify their mathematical structure. *Above all, at each stage of your work on a problem, use the least explicit forms of physical expressions and equations you possibly can.***

- Keep it simple—but not too simple: *At each stage of your solution, use the least explicit expressions you can and still make progress.*
- *Keep your focus on the mathematical structure of the expressions and equations you're using.*
- *Once you've formulated the problem, express everything in terms of dimensionless quantities*, in order to reduce clutter and expose the underlying structure of the expressions in the problem (see Appendix C).
- *At frequent intervals, clear away the algebraic underbrush.* Try to eliminate algebraic clutter (e.g., by introducing auxiliary constants, variables, and functions) so you can see clearly the structure of your expressions.

A.2.7 The Principle of Alert Awareness

I lied. This principle is even more important than the last one. If you follow this alone—and ignore all my other advice—you'll still markedly improve your effectiveness as a problem solver.

> **Stay alert. Keep your brain engaged! Don't let the problem lull you into a mindless stupor! Never solve problems on autopilot.**

Stop frequently to ask yourself questions like the following,

- What do these quantities I'm working with mean physically?
- If I don't know *the* answer, then what *would* an answer to this question be like?
- Is this problem becoming more complicated than I think it should?
- Do my answers or intermediate results make sense? Are they of sensible order of magnitude? Are they dimensionally correct? Do they "feel wrong" somehow?

♦ **Tip:** *Don't forget that your work on a problem isn't through until you've checked* dimensions, orders of magnitude, *and* simple limits*—and anything else you can think of!*

Bibliography